

# Basic Study on Hybrid Systems to Improve the Vehicle Performance of Small Race Car (Investigation of Gear Ratios for Hybrid Systems Using Lap Time Simulation)

by

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## Abstract

To improve the dynamic performance of small hybrid racing vehicles which use an electric motor and internal combustion engine, we propose a hybrid system to synchronize these torques with a simple configuration. The proposed hybrid system is suitable for a small race car: it is lightweight and can be arranged in various layouts with a simple structure using chains and sprockets. To investigate the control system for the hybrid system, it is necessary to analyze the dynamic performance of a vehicle in which the proposed system is installed. Therefore, we constructed a dynamic simulation when the vehicle was driven on a circuit. This paper investigates the motor gear ratio and final gear ratio for optimizing the vehicle performance of the hybrid system in a small race car using lap time simulation. The analysis results show that not only can the optimum setting be obtained by changing the final reduction ratio and the reduction ratio of the motor in the circuit, but also that the optimum value varies with different speed ranges in different circuits.

**Keywords:** Hybrid vehicles, Torque synthesizer, Small race car, Vehicle dynamics, Lap time simulation

## 1. Introduction

To examine the performance of a vehicle, three aspects of performance are evaluated: acceleration, turning, and braking. In recent years, race car, which pursue vehicle performance to the extreme, have begun to use hybrid systems consisting of an internal combustion engine and an electric motor as a power source. Hybrid vehicles, which are powered by two systems, an internal combustion engine and a motor, use the characteristics of the motor to compensate for the lack of torque in the low-speed range of gasoline-powered vehicles, and can be expected to deliver high torque and high acceleration over a wide speed range. Taking advantage of these advantages, recent general vehicles equipped with hybrid systems on the market aim to reduce fuel consumption by downsizing the internal combustion engine. Research on hybrid systems for energy management and comfortable

driving with a primary focus on improving fuel economy has been actively conducted<sup>1,2)</sup>. In race cars, on the other hand, the most important objective is to win the competition. To achieve this objective, it is of utmost importance to improve the dynamic performance of the powertrain, which consists of the engine and electric motor, and to minimize the lap time. In order to drive faster on a defined circuit, the acceleration performance must be improved, especially to reach the maximum speed in a short time. In previous studies, battery optimal control methods for hybrid systems and comparison of acceleration performance for different layouts of two power systems have been reported to minimize lap time<sup>3,4)</sup>, but there have not been enough studies to examine more active use of hybrid systems for driving performance. In addition, these studies have been performed on hybrid systems such as conventional series, parallel, power-split, and series-parallel hybrid powertrains<sup>5)</sup>. These systems use gears or planetary gears to combine the power of the engine and electric motor<sup>6)</sup>, but have drawbacks such as heavy systems and limited parts layout. Therefore, the authors have proposed a hybrid system suitable for small race car, which is lightweight and can be arranged in various layouts with a simple structure using chains and sprockets, and have studied the improvement of dynamic performance of vehicles by hybrid systems<sup>7)</sup>. Lap time simulation is used to predict vehicle performance in a

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given circuit. This is a tool that converts various elements of a vehicle into lap times. There are many types of simulation methods, ranging from simple steady-state simulations to multi-body simulations involving complex transient phenomena<sup>8-11)</sup>. Previous studies have reported the usefulness of quasi-steady-state simulations in terms of computational load and time prediction accuracy<sup>12,13)</sup>. The nonlinear tire model<sup>14)</sup> and the four-wheels model<sup>15)</sup> that considers load transfer have also been applied to the tire characteristics. However, these studies were conducted on vehicles powered only by engines, and there is no sufficient study of the effect of the output characteristics of the hybrid system proposed by the authors on the reduction of running time in a circuit consisting of multiple straight lines and corners with varithesizingous curvatures. Therefore, we used a quasi-steady-state method with a small computational load and high prediction accuracy to calculate the lap time in a given circuit, and studied the dynamic performance of the hybrid system. In this report, the effects of changing the motor gear ratio and final gear ratio of a rear-wheel-drive parallel hybrid system on driving performance were investigated using a simple vehicle model that does not consider nonlinear tire models or load transfer as a basic study and a calculation method for quasi-steady-state lap time simulation.

The novelty of this paper is the establishment of a simulation of the dynamic performance of a vehicle with motor assist for the torque of the engine shifted by the transmission. In many competition vehicles equipped with such hybrid systems, a motorcycle engine with a transmission built into the engine case is used. Few reports have been published on hybrid systems in which an electric motor assists the torque already reduced by the transmission at the engine output shaft. This paper describes a lap performance simulation of such a hybrid system and the calculation of optimal parameters for a given circuit as an actual situation.

## 2. Hybrid Systems for Small Race Car

As the subject of the hybrid system to be installed in the small race car, we referred to Formula Hybrid, the hybrid division of Formula SAE, which is developed worldwide as an educational program for students who want to become automotive engineers, and used the regulations of this competition.

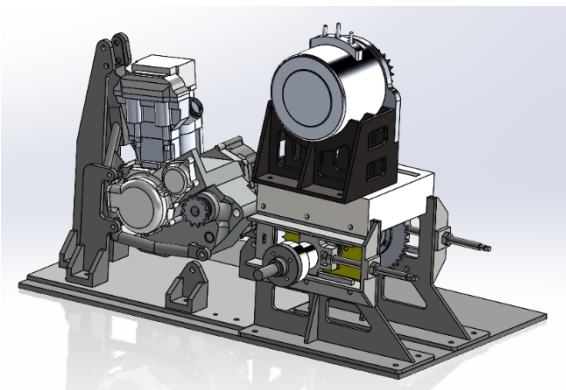


Fig. 1 3D model of the hybrid system.

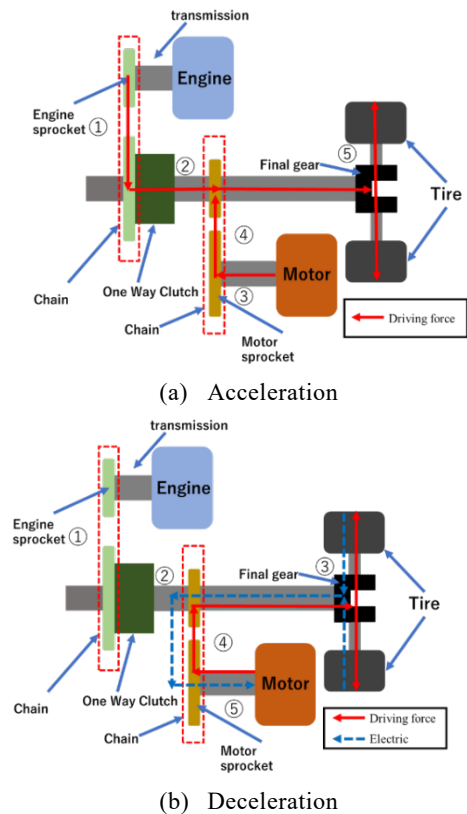


Fig. 2 Proposal of a device for synthesizing traction force generated by internal combustion engine and electric motor.

### 2.1 Structure of the hybrid system

The hybrid system is equipped with a torque synthesizer instead of the planetary gear transmission used in many passenger cars. Figure 1 show 3D model of the proposed hybrid system.

The planetary gear type has many layout restrictions and is difficult to maintain, whereas the torque synthesizer has the advantage of easy maintenance, fewer design requirements on the chassis side, and the ability to use a chain, making it easy to layout. This makes it suitable for hybridization of motorcycle engines.



Fig. 3 Internal combustion engine YAMAHA G363E.

Table 1 Specifications of the engine.

|                     |                                              |
|---------------------|----------------------------------------------|
| Engine Model Number | G363E                                        |
| Engine Type         | 250 cc liquid-cooled DOHC 4-stroke; 4 valves |
| Bore × Stroke       | 77.0 mm × 53.6 mm                            |
| Compression Ratio   | 11.8:1                                       |
| Fuel Delivery       | Fuel injection                               |
| Ignition            | TCI: Transistor Controlled Ignition          |
| Transmission        | Constant-mesh 6-speed, multiplate wet clutch |
| Max Power           | 23 kW(35 PS)/10000 rpm                       |
| Max Torque          | 24 N · m(2.2 kg · m)/8000 rpm                |

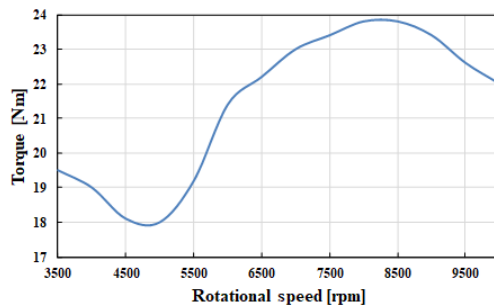


Fig. 4 Relationship between engine rotational speed and torque.

Figure 2 shows schematic diagram of proposed hybrid system. As shown in Fig. 2(a), in the proposed hybrid system, during acceleration, the shaft torque generated by the engine is shifted through the transmission, decelerated and transmitted from the engine sprocket to the drive shaft side sprocket by a chain, and then transmitted to the axle shaft through the one-way clutch and final gear. The shaft torque generated by the motor is decelerated and transmitted to the drive shaft side sprocket by a chain, and then transmitted to the axle by the final gear. The drive torque generated around the tire is the sum of the above two torques transmitted to the axle shafts

On the other hand, during deceleration, as shown in Fig. 2(b), there is no drive power from the engine and the motor drive power is not transmitted to the engine via the one-way clutch, and the engine sprocket on the drive shaft side idles. In



Fig. 5 Electric motor Moterenergy ME0913.

Table 2 Specifications of the motor.

|                           |                               |
|---------------------------|-------------------------------|
| Drive voltage             | DC24 V - 96 V                 |
| Maximum current           | 550 A                         |
| Maximum rotation speed    | 5000 rpm (No load)            |
| Maximum output            | 30 kw (DC 96 V)               |
| Maximum torque            | 90 Nm                         |
| Continuous output         | 12 kw (DC 96 V)               |
| Continuous current        | 180 A                         |
| Continuous efficiency     | 92%                           |
| Continuous rotation speed | 3000 rpm                      |
| Weight                    | 15.9 kg                       |
| Poles                     | 4 (Magnet 8)                  |
| Cooling method            | Air cooling Forced fan system |

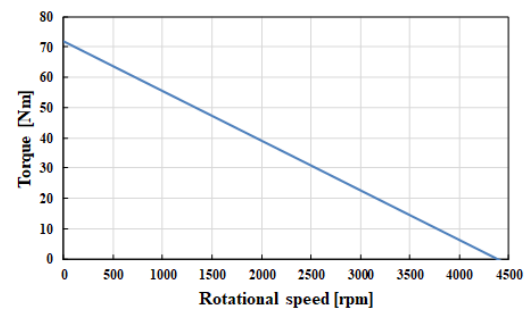


Fig. 6 Relationship between motor rotational speed and torque.

addition, the braking force on the tire contact patch generates torque on the axle shaft in the opposite direction of acceleration, which is transmitted to the motor shaft by the final gear and sprockets. These torques can generate power to the motor and regenerate energy.

## 2.2 Two power sources in a hybrid system

The internal combustion engine in the hybrid system is the G363E used in the YAMAHA WR250R shown in Fig. 3. Table 1 shows the specifications of the engine, and Fig. 4 shows the two-dimensional data of relationship between rotational speed and torque.

The electric motor in the hybrid system is specified to have a maximum output of less than 30 kW and uses the air-cooled Motenergy ME0913 brushless motor shown in Fig. 5. Table 2 shows the specifications of the motor, and Fig. 6 shows the relationship between rotational speed and torque.

The torque relative to the rotational speed of these two power sources is synthesized using the values of the respective gear ratios in the engine, the engine and motor gear ratios up to the drive shaft, and the final gear ratio from the drive shaft to the axle shafts.

## 3. Calculation Method of Quasi-Steady-State Lap Time Simulation

In this simulation, the vehicle is always assumed to be in equilibrium and yaw motion is ignored. The velocity in each section is determined as the maximum velocity that can be achieved when driving within the range of a three-dimensional map consisting of lateral acceleration, longitudinal

acceleration, and speed, called the GGV vehicle performance envelope, which is pre-calculated from the vehicle model. Figure 7 shows a schematic of the GGV vehicle performance envelope. It can be read that the performance of longitudinal acceleration and lateral acceleration increases or decreases depending on the size of the velocity. In general, changes in velocity affect the vehicle's friction ellipse due to aerodynamics and engine speed. Lap time is calculated by discretizing the times for each segment and summing the times for all segments.

### 3.1 Simplified vehicle model

The lateral and longitudinal forces  $F_{x,y}$  at the tire contact patch are calculated from the simplified linear tire model shown in eq. (1) by multiplying the vertical load  $F_z$  by the friction coefficients in the lateral and longitudinal directions  $\mu_{x,y}$ .

The vertical load is the weight produced by the vehicle mass  $m$  and the acceleration of gravity  $g$  and the aerodynamic function downforce  $L$ , which is calculated by eq. (2). The downforce is then calculated in eq. (3) and is found to be a function of speed  $v$ .

$$F_{x,y} = F_z \cdot \mu_{x,y} \quad (1)$$

$$F_z = mg + L \quad (2)$$

$$L = 0.5 \cdot C_L A \rho v^2 \quad (3)$$

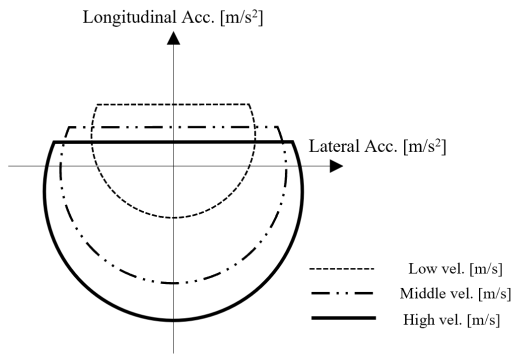


Fig. 7 Schematic of the GGV vehicle performance envelope.

The tractive force  $F_t$  derived from the hybrid powertrain system is calculated from the vehicle speed  $v$ , each gear ratio  $gr$ , and tire radius  $r_t$  using eq. (4) to calculate the engine speed and motor speed  $\omega$ , respectively, and the shaft torque  $T$  of the two systems from these rotational speeds are added together and calculated by eq. (5) using the tire radius.

$$\omega = \frac{v \cdot gr}{r_t} \quad (4)$$

$$F_t = \frac{T \cdot gr}{r_t} \quad (5)$$

The lower of the tire longitudinal force and the powertrain-derived driving force obtained in eqs. (1) and (5) is used as the longitudinal force at the vehicle center of gravity. This is because the longitudinal force at the tire contact patch is highly dependent on the slip ratio, which is the ratio of the vehicle speed to the rotational speed of the tire, and the drive torque generates the tire rotational speed, which varies only slightly, around 0 - 0.2, hence the approximation in eq. (6) is used. Longitudinal forces at the vehicle center of gravity can be calculated by allowing the drag force  $D$  of the aerodynamic function to be taken into account. Drag force is a function of speed, as is downforce in eq. (3), and can be calculated in eq. (7) using the drag coefficient  $C_L$ .

$$T \approx F_x \cdot r_t \quad (6)$$

$$D = 0.5 \cdot C_D A \rho v^2 \quad (7)$$

With the calculations up to this point, it is possible to calculate the maximum lateral and longitudinal forces for each velocity. In order to calculate the combined state in which these longitudinal and lateral forces occur simultaneously in the quasi-steady-state, the friction ellipse assumption in eq. (8) is made to simplify the problem.

$$(\mu_{x,y} \cdot F_z)^2 = F_x^2 + F_y^2 \quad (8)$$

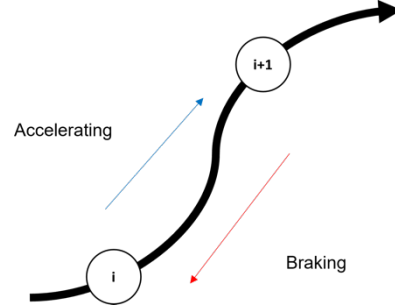


Fig. 8 Illustration of quasi-steady-state simulation.

### 3.2 Calculation method for lap time in quasi-steady-state

Quasi-steady-state lap time simulation in this research is calculated from pre-discretized circuit data of radius of curvature and distance. These can be obtained from GPS and data logging systems installed in the actual vehicles.

First, a critical speed profile is created in eq. (9) using the corner radius  $R$  and the maximum lateral acceleration  $a_y$  calculated from the vehicle model. At this time, the velocity within each section is not within the GGV vehicle performance envelope when longitudinal acceleration occurs due to the maximum lateral acceleration. To solve this problem, the critical speed profile is used to calculate the longitudinal acceleration  $a_x$  of the velocity  $v$  within the adjacent section distance  $dx$  from the beginning and end of the circuit using eq. (10), and the acceleration and deceleration velocities are calculated by decreasing the velocity until it falls within the GGV vehicle performance envelope. An illustration of the

calculation of acceleration side velocity and deceleration side velocity within an adjacent section is shown in Fig. 8.

$$a_y = \ddot{y} = \frac{v^2}{R} \quad (9)$$

$$\left. \begin{aligned} dv &= v_i - v_{i-1} \\ dt &= \frac{dx}{v_i} \\ a_x &= \ddot{x} = \frac{dv}{dt} \end{aligned} \right\} \quad (10)$$

From these calculations, the intersection of the two velocities, the acceleration side velocity and the deceleration side velocity with respect to distance, is calculated as the braking start point, and the limit performance of the vehicle in the quasi-steady-state can be predicted.

The computation run time is very fast; a single lap simulation takes less than 30 seconds. This makes it possible to analyze the sensitivity of the lap times when various parameters are changed.

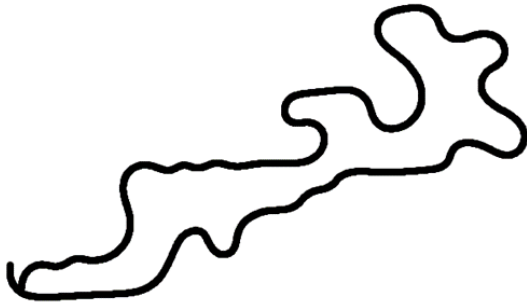


Fig. 9 Schematic of evaluation 900-meter circuit.

Table 3 Vehicle basic parameters.

| General Data                     |       | Engine Data            |       |
|----------------------------------|-------|------------------------|-------|
| Vehicle Type                     | FSAE  | Thermal Efficiency [%] | 35    |
| Mass [kg]                        | 369.7 | Transmission Data      |       |
| Driven Type                      | 2WD   | Gear1                  | 2.642 |
| Aero Data                        |       | Gear2                  | 1.812 |
| Drag Coefficient                 | 0.8   | Gear3                  | 1.318 |
| Downforce Coefficient            | 1.2   | Gear4                  | 1.04  |
| Front Area [m <sup>2</sup> ]     | 1.1   | Gear5                  | 0.888 |
| Air Density [kg/m <sup>3</sup> ] | 1.226 | Gear6                  | 0.785 |
| Tire Data                        |       | Final Drive Ratio      | 3.145 |
| Tire Radius [m]                  | 0.128 | Drive Efficiency [%]   | 83    |
| Rolling Resistance               | 0.03  |                        |       |
| Longitudinal Friction            | 1.4   |                        |       |
| Lateral Friction                 | 1.5   |                        |       |

#### 4. Final Gear Ratio and Motor Reduction Gear Ratio Study Using Lap Time Simulation

##### 4.1 Simulation Calculation Conditions

In order to evaluate the vehicle performance of the proposed small race car, the power supply voltage of the electric motor is 77 V, based on the Table 2. A reduction

gearbox was added to the motor, the engine and motor speeds were individualised, a program was created to allow the torque and speed of the motor to be varied, and the reduction ratio of the motor and final gear ratio were changed for lap time simulations.

The primary reduction ratio of the engine was set to 3.12, and no lower than 3500 rpm was used in this study because the engine would stop at low engine speeds when starting the vehicle. Therefore, the initial speed at start-up was set in first gear and at an engine speed of 3500 rpm. The engine speed is shifted up when the engine speed reaches 10000 rpm, and the value at 6th gear and 10000 rpm is held constant in the simulation after 6th gear and 10000 rpm. The engine and

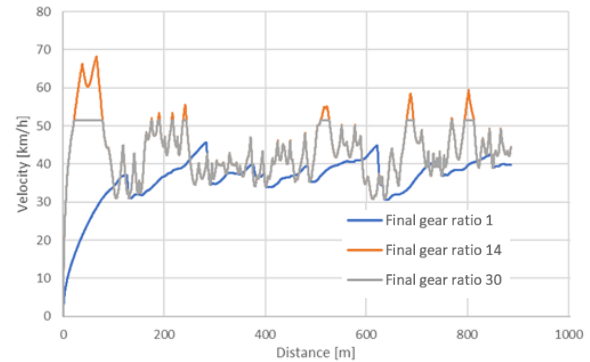


Fig. 10 Relationship between distance and vehicle velocity for each final gear ratio.

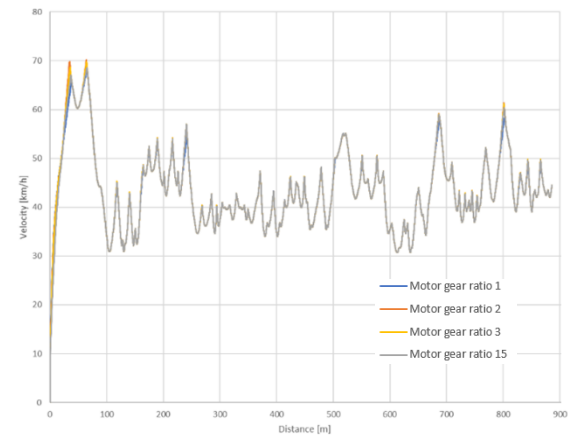


Fig. 11 Relationship between distance and vehicle velocity for each electric motor reduction gear ratio.

motor shaft torques were modeled by polynomial approximation using the two-dimensional data in Fig. 4 and 6. For the other basic parameters, the simplified vehicle model in Table 3 was used. The evaluation circuit was a 900 m closed circuit with a series of left and right turns, as shown in Fig. 9. These are the circuits used for Endurance Event in Formula

SAE Japan<sup>16)</sup> which is based on FORMULA SAE Rules 2022<sup>17)</sup> defined by Society of Automotive Engineers International, a project similar to the vehicle proposed in this study.

#### 4.2 Analysis Result

Simulations were carried out on the small hybrid race car with varying final and motor reduction gear ratios. We aimed to minimize the lap time in order to validate the effective vehicle state for the purpose of predicting the scale of the vehicle state to be achieved on the course in order to construct a multi-input, multi-output control algorithm to be installed in the proposed vehicle. The objective is to minimise lap times and the role of the powertrain is to achieve the maximum possible acceleration in each velocity range. There is a trade-off between lowering the final gear ratio, which reduces the

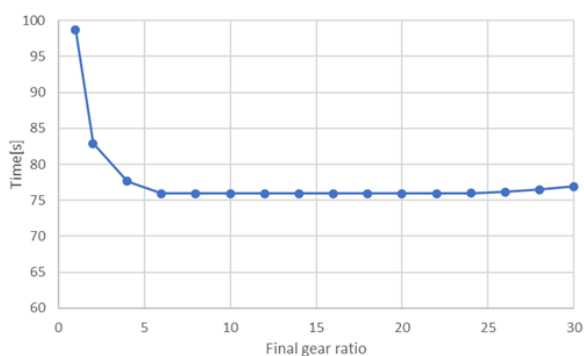


Fig. 12 Relationship between final gear ratio and lap time.

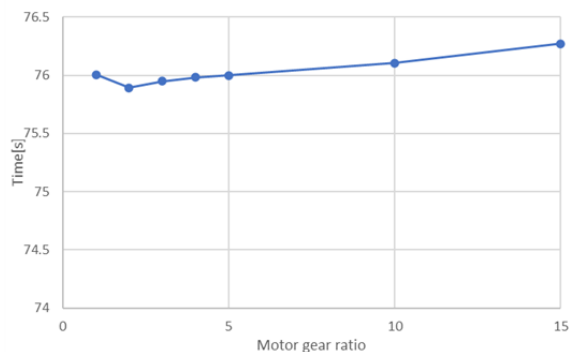


Fig. 13 Relationship between electric motor reduction gear ratio and lap time.

acceleration that can be generated due to the reduction in axle torque, and raising it too high, which reduces the maximum speed due to power unit speed limitations, and the tyres reaching their slip limit, resulting in excess powertrain dynamic performance. Figure 10 shows analyzed results in final gear ratio 1, 14, 30. This figure shows that when the final gear ratio is 1, the overall slope of speed is low, and when the final gear ratio is 30, the speed has hit its head. In addition, the slope of the speed is always the same when comparing final gear ratios 14 and 30, indicating that an excessive gear ratio exceeds the limit of the longitudinal force of the tyres, resulting in a wasteful use of powertrain dynamic performance.

The reduction ratio of the motor has a smaller range of gear ratio settings, as the motor can generate more torque at

lower speeds due to the characteristics of the motor. The analyzed relationship between distance and velocity for each reduction ratio of the motor is shown in Fig. 11. The speed range where the characteristics of the motor can be utilised is the high velocity range, and the difference between each gear ratio appears at the point where the highest velocity is reached in one lap. Extremely large motor gear ratios exceed the limits of the longitudinal force of the tyres, and the advantages of the hybrid system disappear.

The relationship between final gear ratio and obtained lap time is shown in Fig. 12, and the relationship between motor reduction gear ratio and running time is shown in Fig. 13. The simulations on this evaluation circuit produced the fastest results when the final gear ratio was 14 and the motor reduction gear ratio was 2.

#### 5. Conclusion

This paper presents a fundamental study of the motor gear ratio and final gear ratio for optimizing the vehicle performance of the proposed hybrid system in a small race car, using a simulation-based approach. Specifically, as a first step to predict the scale of the vehicle state to be achieved on the course, such as the speed of the vehicle to construct a multi-input multi-output control algorithm for the proposed racing car, we found the optimal solution of the drive gear ratio for the racing car to predict the effective vehicle state for the competition.

Within the report, the superiority of the hybrid system using the torque synthesizer mechanism was asserted in comparison with the planetary gear mechanism, and the modeling method along this mechanism was briefly presented. And, the theory and utility of quasi-steady state lap time simulation, which models the vehicle and predicts its performance in a simplified manner, as a powerful tool to help optimize powertrain systems, is presented. By extending the model, more practical simulations can be expected, and parameter sensitivity analysis can be performed outside of this research.

In the future, based on the analysis results, we will proceed with the development and design of the actual vehicle, as well as simulation of closed-loop circuit driving. We will also develop a vehicle dynamics simulation tool that includes more detailed nonlinear elements in order to predict complex vehicle motion phenomena that occur when the vehicle is actually driven and to build control algorithms.

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