

# Energy Harvesting Characteristics of Electricity Generation System with Piezoelectric Element under Random Excitation

by

Hiroyuki MORIYAMA<sup>\*1</sup>, Hirotarou TSUCHIYA<sup>\*2</sup> and Helmut Takahiro UCHIDA<sup>\*3</sup>

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## Abstract

In this study, a circular plate that is installing a piezoelectric element at its center is adopted as energy-harvesting system and is subjected to a harmonic point force. To assume the application of this electricity generation system to actual environment, the point force, whose frequency is different from the natural frequency, is added to the harmonic point force. Then the excitation force ratio, which is determined by dividing the additional point force by the original point force, is defined. The mechanical power supplied to the plate, the electric power in the electricity generation and the electricity generation efficiency derived from those powers are considered as electricity generation characteristics from theoretical analysis under such an excitation condition. The additional excitation weakens rapidly these electricity generation characteristics, as its excitation frequency goes away from the natural frequency. It is verified that such variations in electricity generation characteristics get closer to the situation of the random excitation from the electricity generation experiment. Then from the estimation based on  $Q$  value, the electricity generation efficiency becomes more insensitive against shifts in the additional excitation frequency than the mechanical power and electric power and also becomes insensitive with increases in the plate thickness and excitation force ratio. To improve the electricity generation characteristics by using the acoustic radiation power derived from the plate vibration, a cylinder that has the above plates at both ends is adopted, and the mechanical-acoustic coupling between the plate vibrations and internal sound field is used. However, although the application of the random excitation to this system suppresses the electricity generation characteristics more extremely than those of the harmonic excitation, the characteristics are hardly improved only by mechanical-acoustic coupling.

**Keywords:** Energy harvesting, Electricity generation system, Piezoelectric element, Harmonic excitation, Random excitation

## 1. Introduction

Energy harvesting is an energy conversion technology with an output power of  $\mu\text{W}$  to  $\text{mW}$ , or at most a few  $\text{W}$ , which can be used as a stand-alone power source for small electronic devices<sup>1)</sup>. Generally, it utilizes various energy sources, such as heat, wind, solar energies, and so on. Among the various energy harvesting possibilities being proposed and under consideration for practical application, the energy harvesting technologies based on vibration power generation will become increasingly essential as a power source technology in the future<sup>2,3)</sup>, including concepts such as the "Trillion Sensor," which will create a market of 1 trillion sensors per year<sup>4)</sup>. This power generation requires a two-step process: first, the

mechanical energy in the environment is captured in the device, and subsequently converted into electrical energy. From a material point of view, there are two types of vibration power generation: magnetostrictive and piezoelectric. In both cases, it is important to model and understand the vibration problem according to the realistic environment, for a better energy conversion efficiency<sup>5)</sup>. The Sustainable Development Goals (SDGs), proposed by the United Nations in 2015<sup>6)</sup> call for improvements in global energy efficiency by 2030, and improvements in the harvesting efficiency by understanding vibration will make a significant contribution to this goal. Especially, untapped vibration energy exists fully around us and technologies to be able to scavenge such an energy are so significant. Although several methods to harvest the vibration energy have been proposed a lot, piezoelectric materials that can convert it into usable electric energy are taken up and electricity generation systems that are attaching the materials

<sup>\*1</sup> Professor, Department of Prime Mover Engineering

<sup>\*2</sup> Assistant Professor, Department of Precision Engineering

<sup>\*3</sup> Junior Associate Professor, Department of Precision Engineering

to host structures attract considerable attention. Because a simple cantilever beam is adopted generally as the host structure, it is comparatively easy to theoretically model it and to carry out the theoretical procedure model<sup>7)</sup>.

In such an energy-harvesting system, matching the load impedance with the input impedance is significant. Therefore, the variations in the maximum power have been considered by changing the resistance of the resistor by which the load impedance was characterized<sup>8,10)</sup>. Then to improve the modal equivalent stiffness ratio, modal electro-mechanical coupling coefficient, and modal piezoelectric voltage coefficient of a piezoelectric element attached to a beam, two mechanical impedance matching methods were proposed, and then they were derived from using spacers between the piezoelectric element and beam structure and from tuning for the size of the piezoelectric element<sup>9)</sup>. In the case of the cantilever beam that was selected as the host structure, because the vibration characteristics were adjusted easily due to a tip mass and related directly to the efficiency (with respect to the tip mass), the influence of the size and position on electricity generation characteristics was considered<sup>8)</sup>. In general, the rectangular strip shape is adopted as the host structure. However, because it is also possible to assume various shapes, electricity generation characteristics were compared in rectangular and triangular shapes, and it was verified that the performance of a triangular shape was superior to that of a rectangular shape<sup>10)</sup>.

In this study, a circular plate, on which a piezoelectric element is installed at its center, is adopted as the host structure and is subjected to a harmonic point force. However, to assume the application of this electricity generation system to actual environment, the point force, whose frequency is different from the natural frequency, is also added to the original point force. Then the excitation force ratio, which is determined by dividing the additional point force by the original point force, is defined. The mechanical power supplied to the plate, the electric power in the electricity generation and the electricity generation efficiency derived from those powers are considered as electricity generation characteristics from theoretical analysis under such an excitation condition. On the other hand, to utilize an acoustic radiation derived from the plate vibration for energy-harvesting, a cylinder that has the above plates at both ends is also adopted as the electricity generation system and mechanical-acoustic coupling is caused between the plate vibrations and a sound field into the cylindrical enclosure by subjecting one side of each plate to harmonic or random excitation. Then the effect of coupling is evaluated by comparing with the efficiencies in the electricity generation system of only plate.

## 2. Analytical method

### 2.1 Analytical model

Fig. 1 shows the analytical model of a plate that is supported by translational and rotational springs. These springs are distributed at constant intervals and the support conditions are determined by the translational spring stiffness  $T$  and the rotational spring stiffness  $R$ . The plate whose radius and thickness are denoted by  $r_c$  and  $h_c$ , respectively, have a Young's modulus  $E_c$  and a Poisson's ratio  $\nu_c$ . Piezoelectric element is installed at the center of the plates and have radius  $r_p$ , thickness  $h_p$ , Young's modulus  $E_p$ , and Poisson's ratio  $\nu_p$ . Then an electrode plate that is made of brass is sandwiched between the above plate and piezoelectric element and has radius  $r_b$ , thickness  $h_b$ , Young's modulus  $E_b$  and Poisson's ratio  $\nu_b$ . The suffixes  $c$ ,  $p$  and  $b$  denote the circular plate, piezoelectric element and electrode plate. The coordinates are radius  $r$  and angle  $\theta$  on the plate and  $z$  indicates the direction vertical to its surface. The plate was excited by the periodic point force  $F$  at distance  $r_e$  and angle  $\theta_e$  after the  $r\theta$  coordinates and its natural frequency was employed as the excitation frequency. However, the excitations combined with other frequency components were also assumed.

The flexural displacement of the plate is denoted by  $w_c$  and the displacement of the piezoelectric element installed on the plate is denoted by  $w_p$ . Their displacements are expressed by Eq. (1) as suitable trial functions and have the plate mode  $X_{nm}^s$  of Eq. (2).  $w_p$  is identical to  $w_c$ , because it is assumed that the piezoelectric element adheres completely to the plate through the electrode plate.

$$w_c = w_p = \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} X_{nm}^s A_{nm}^s (e^{j\omega_1 t} + e^{j\omega_2 t} + \cdots + e^{j\omega_k t}), \quad k = 1, 2, \cdots, \quad (1)$$

$$X_{nm}^s = \sin(n\theta + \pi/2) (r/r_c)^m, \quad (2)$$

where  $n$  and  $m$  are the circumferential and radial orders and  $s$  is symmetry index with respect to the vibration mode.  $A_{nm}^s$  is coefficient to be determined and correspond to the amplitude of the flexural displacement.  $\omega_1$  is the angular frequency in the

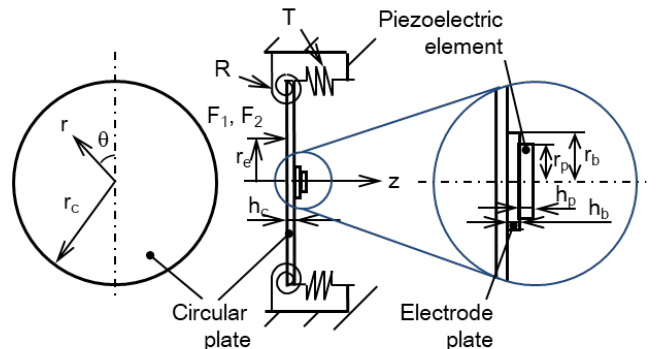


Fig. 1 Configuration of analytical model.

natural mode (0,0), which is denoted by  $(n,m)$  mode, and corresponds to the angular frequency of the harmonic point force acting on the plate. Therefore,  $\omega_2, \omega_3, \dots, \omega_k$  correspond to the angular frequencies of point forces added to the harmonic point force and  $t$  is the elapsed time.

With respect to the piezoelectric element on the plate, the electric field  $E_p$ , which occurs in the above direction of the piezoelectric element, is expressed as follows:

$$E_p = Y_{nm} v_p = -R_p \dot{q}_p \\ = j\omega B_{nm}^s \{e^{j\omega_1 t} + e^{j\omega_2 t} + \dots + e^{j\omega_k t}\}. \quad (3)$$

$v_p$  is the voltage that occurs in the electric field. The electric potential across the piezoelectric element is constant since it is assumed that it does not reach the plate, so that  $Y_{nm}$  is defined as Equation (4).  $R_p$  is the overall resistance value in an electricity generation circuit. The magnitude of the electric charge  $q_p$  depends on the coefficient  $B_{nm}^s$  that is determined in this analysis as with  $A_{nm}^s$ .

$$Y_{nm} = \begin{cases} -1/h_p, & h_c/2 + h_b < z < h_c/2 + h_b + h_p, \\ 0, & -h_c/2 < z < h_c/2 + h_p, \end{cases} \quad (4)$$

$$q_p = B_{nm}^s \{e^{j\omega_1 t} + e^{j\omega_2 t} + \dots + e^{j\omega_k t}\}. \quad (5)$$

To consider electro-mechanical coupling between the plate and piezoelectric element, the elements  $M_{nmm'}^s$  and  $K_{nmm'}^s$  of the mass and stiffness matrices can be defined as

$$M_{pnm}^s = \int_{V_p} \rho_p X_{nm}^s X_{nm'}^s dV_p, \quad (6)$$

$$K_{pnm}^s = \int_{V_p} z^2 X_{nm}^s \text{"} E_p^E X_{nm'}^s \text{"} dV_p. \quad (7)$$

The index  $m'$  has a transposed relation to a radial order  $m$ , then  $\rho_p$  and  $V_p$  denote the density and volume of the piezoelectric element, respectively. The respective elements of electro-mechanical coupling and capacitance matrices are denoted by  $\beta$  and  $C_p$  and they are defined as follows:

$$\beta = - \int_{V_p} z \rho_p X_{nm}^s e Y_{nm'} dV_p, \quad (8)$$

$$C_p = \int_{V_p} Y_{nm} \gamma^E Y_{nm'} dV_p, \quad (9)$$

$$e = d_{31} E_p^E, \quad (10)$$

$e$  is the piezoelectric coupling coefficient that includes the piezoelectric strain constant  $d_{31}$ , as described in Equation (10) and  $\gamma^E$  indicates the dielectric constant when a strain is constant.

## 2.2 Governing equations of electro-mechanical coupling

To formulate the plate motion, Hamilton's principle is applied to the analytical model:

$$\delta H = \delta \int_{t_0}^{t_1} (T_c + T_p + T_b - U_c - U_p - U_b - U_s + W) dt = 0, \quad (11)$$

where  $H$  is the Hamiltonian;  $T_c$ ,  $T_p$  and  $T_b$  are the kinetic energy of the circular plate, piezoelectric element and electrode plate, respectively; and  $U_c$ ,  $U_p$  and  $U_b$  are their respective potential energies.  $U_s$  is the elastic energy stored in the springs, and  $W$  is the total work done on the plate by the point force. Finally,  $t_0$  and  $t_1$  are two arbitrary times. On the other hand, if the first variation is carried out in terms of the piezoelectric part on the plate, we can obtain

$$\delta \int_{t_0}^{t_1} (T_p - U_p) dt = \frac{1}{2} \int_{t_0}^{t_1} \int_{V_p} \{\rho_p \dot{w}_p \delta \dot{w}_p - (E_p^E \varepsilon_p - e E_p) \delta \varepsilon_p + (e \varepsilon_p + \gamma^E E_p) \delta E_p\} V_p dt, \quad (12)$$

where  $\dot{w}_p$  is the vibration velocity of the piezoelectric element on the plate and  $\varepsilon_p$  is the in-plane strain within that element.

Here, the governing equation of electro-mechanical coupling with respect to the system is determined. This equation defines the mechanical motion and electrical characteristics of this coupling system. The motion equation of the plate was obtained by applying the flexural displacement  $w_c$  of Eq. (1) to the Hamiltonian  $H$  of Eq. (11) in the same manner as Eq. (12) of the piezoelectric part. Then the motion of the plate having a piezoelectric part, which generates electricity by the plate vibration, is governed by the following Eq. (13):

$$\sum_{m'=0}^{\infty} \{ [K_{cnmm'}^s (1 + j\eta_c) + K_{pnm}^s (1 + j\eta_p) + K_{bnmm'}^s \times \\ K_{bnmm'}^s (1 + j\eta_b) + r_c F_{sn} \left\{ T + \left( \frac{m}{r_c} \right) \left( \frac{m'}{r_c} \right) R \right\} ] \{ e^{j(\omega_1 t + \alpha_1)} + \\ e^{j(\omega_2 t + \alpha_2)} + \dots + e^{j(\omega_k t + \alpha_k)} \} - (M_{cnmm'}^s + M_{pnm}^s + \\ M_{bnmm'}^s) \{ \omega_1^2 e^{j(\omega_1 t + \alpha_1)} + \omega_2^2 e^{j(\omega_2 t + \alpha_2)} + \dots + \\ \omega_k^2 e^{j(\omega_k t + \alpha_k)} \} \} A_{nm}^s - \sum_{m'=0}^{\infty} \beta v_p B_{nm'}^s \{ e^{j(\omega_1 t + \alpha_1)} + \\ e^{j(\omega_2 t + \alpha_2)} + \dots + e^{j(\omega_k t + \alpha_k)} \} \\ = \mathbf{F}_{nm}^s \{ e^{j(\omega_1 t + \alpha_1)} + e^{j(\omega_2 t + \alpha_2)} + \dots + e^{j(\omega_k t + \alpha_k)} \}. \quad (13)$$

$K_{cnmm'}^s$  and  $K_{bnmm'}^s$  are stiffness matrix elements of the structural and electrode plates, respectively.  $M_{cnmm'}^s$  and  $M_{bnmm'}^s$  are also mass matrix elements of the respective parts. These are elements of the symmetrical matrices, since the index  $m'$  has a transposed relation to  $m$ , as with  $M_{pnm}^s$  and  $K_{pnm}^s$ .  $\eta_c$ ,  $\eta_p$  and  $\eta_b$  signify the structural damping factors of the end plate, piezoelectric element and electrode plate, respectively. Then  $F_{sn}$  is a coefficient that is determined by the indices  $n$  and  $s$  and  $\mathbf{F}_{nm}^s$  is a load vector that is applied to the plate as the point force. The details of the coefficient  $F_{sn}$  and the element  $F_{nm}^s$  of the vector are as follows:

$$F_{sn} = \begin{cases} \pi, & \text{at } n \neq 0, \\ 0, & \text{at } n = 0 \text{ and } s = 0, \\ 2\pi, & \text{at } n = 0 \text{ and } s = 1, \end{cases} \quad (14)$$

$$F_{nm}^s = \int_{A_c} F \delta(r - r_e) \delta(\theta - \theta_e) X_{nm}^s dA_c. \quad (15)$$

Here, the point force is expressed by performing Eq. (15) including the delta function  $\delta$  on the plate, whose area is denoted by  $A_c$ .

On the other hand, the electricity generation behavior of the piezoelectric element is also governed by the following Equations (16):

$$\sum_{m'=0}^{\infty} C_p^{-1} \beta A_{nm'}^s = \sum_{m'=0}^{\infty} (j\omega R_p + C_p^{-1}) B_{nm'}^s. \quad (16)$$

The relationship between  $A_{nm}^s$  and  $B_{nm}^s$  is applied to Eq. (13) and these coefficients can be derived by solving the above simultaneous equations. However, if the plate is excited by the point force of its natural frequency and the behavior is assumed to be harmonic,  $e^{j\omega t}$  could be eliminated in Eq. (13).

Since the flexural displacement  $w_c$  is calculated by substituting  $A_{nm}^s$  for Eq. (1), the vibration velocity  $\dot{w}_c$  is derived from the following equation:

$$\dot{w}_c = j\omega \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} X_{nm}^s A_{nm}^s e^{j\omega t}. \quad (17)$$

The mechanical power  $P_m$  supplied to the plate is obtained from the time history data of the products of the vibration velocity and the force at the excited point. The voltage  $v_p$  is calculated by substituting  $B_{nm}^s$  for Equations (3) and (5), when the piezoelectric element generates electricity by the plate vibration. Then the electric power  $P_e$  is also obtained from the time history data of the products of its voltage  $v_p$  and the electric current corresponding to  $v_p$ .

### 3. Experimental apparatus and method

In the experimental study, a circular aluminum plate was employed and was sandwiched between two flanges fastened by bolts to emulate the analytical model, as shown in Fig. 2(a). The flanges have a short steel cylinder, respectively, to fix the plate on their end faces. Although the cylinder on the excitation side is very short not to prevent the installation of a small vibrator and is similar to a ring, another cylinder have somewhat longer length to disturb an inflow of the acoustic radiation from the excitation side. The plate had the radius  $r_c$  of 153 mm and the thickness  $h_c$  was changed as in 2.0, 2.5, 3.0 and 4.0 mm. The point force was applied to the plate by the small vibrator, the amplitude  $F$  was set to 1 N. The excitation was carried out not only near the natural frequency of the (0,0) mode but also by a random wave. Then the position of the point force  $r_e$  was 61 mm and was set to  $r_e/r_c = 0.4$  by means

of normalizing by radius  $r_c$ .

The point force  $F$  was measured by the load cell embedded with the excitation stick, which was short as much as possible between the plate and load cell to avoid the influence of its additional mass. The acceleration  $a_e$  was measured by the acceleration sensor that was installed in close proximity of the excitation position. Because  $F$  and  $a_e$  were time history data, the vibration velocity  $\dot{w}_c$  was calculated from  $a_e$ . The mechanical power  $P_m$  supplied to the plate by the small vibrator was estimated from the relationship between  $F$  and  $\dot{w}_c$ .

The piezoelectric element used to perform the electricity generation experiment was comprised of a circular piezoelectric part constructed of ceramics and a circular electrode part constructed of brass. The piezoelectric and ceramics parts had radiuses  $r_p$  and  $r_b$  of 12.5 and 17.5 mm and thicknesses  $h_p$  and  $h_b$  of 0.23 and 0.30 mm, respectively. The piezoelectric element was installed at the center of the plate as with the analytical model. The voltage  $v_p$ , which was generated with the expansion and contraction of the piezoelectric element on the plate surface derived from the flexural vibration of the plate, was measured by FFT analyzer and was also time history data.

The piezoelectric element was incorporated in the resistance circuit, which was composed of three resistors having resistances  $R_v$ ,  $R_i$ , and  $R_c$ .  $R_v$  and  $R_i$  are the resistances of the voltmeter and ammeter, which are incorporated into the power meter 9, and were 2 M $\Omega$  and 2 m $\Omega$ , respectively. However,  $R_c$  is the resistance of the resistor to consume the generated electric power and was mainly 97.5 k $\Omega$ , and then the resistor was connected outside the power meter. Because the piezoelectric element is assumed to be incorporated in the electrical circuit of the energy-harvesting device and to be connected in series with the resistance for power consumption, it is natural that an electrical current is in-phase to the voltage.

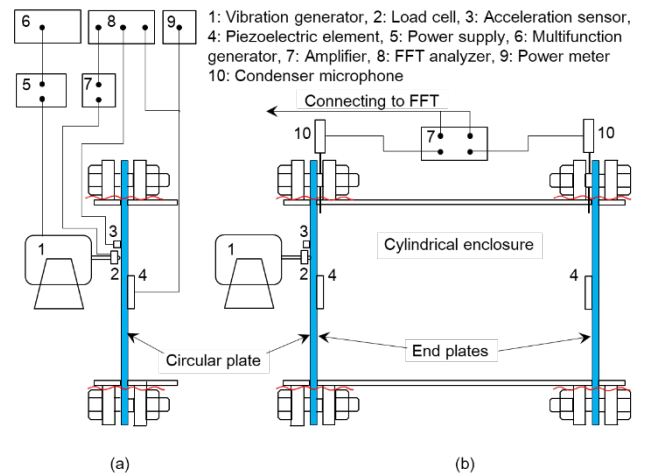


Fig. 2 Configuration of experimental apparatus:  
(a) without cylinder and (b) with cylinder.

Therefore, the electric power  $P_e$  was proportional to the square of  $v_p$  and could be obtained from only the voltage because of Ohm's law. Just in case,  $P_e$  was also measured by the power meter as well.

To utilize an acoustic energy radiated from the plate because of the improvement of the electricity generation performance, a steel cylinder and the same plate were added in the above electricity generation system and this was constructed of three systems, which were the excited plate, cylindrical enclosure and non-excited plate, as shown in Fig. 2(b). The application of mechanical-acoustic coupling, which is caused between the plate vibrations and an internal sound field into the cylindrical enclosure by subjecting one side of each plate to a harmonic point force, is expected to the improvement of the electricity generation performance. The coupling situation was grasped from the estimation of the internal acoustic characteristics, which was involved directly in coupling. Therefore, although the sound pressure level in the cavity was measured using condenser microphones, a probe tube was attached to each microphone not to prevent the sound wave propagation inside the cavity. Because the sound pressure level is maximized near both end plates when the sound field becomes resonant, the tips of the probe tubes were located near the plates and the cylinder wall. The measuring items except the sound pressure level were carried out in the same manner as those of the electricity generation experiment of only the plate.

#### 4. Results and discussion

##### 4.1 Influence of plate thickness and support condition electricity generation characteristics

The plate and piezoelectric element were the same as the experimental apparatus in the respective dimensions, and had the mechanical and electrical properties shown in Table 1. The plate radii  $r_c$  were constant at 153 mm, and the plate thickness  $h_c$  ranged from 1.0 to 10 mm. The analysis herein was based on the assumption of a thin circular plate, specifically, the stress was assumed as the plane stress field, so that the stress was not considered in the direction normal to the plane. Therefore, because the stress was increased relatively in the plate thickness direction with increasing  $h_c$ , it was supposed that the deviation between the analytical and actual situations

Table 1 Mechanical and electrical properties.

|                                                                                | Density [kg/m <sup>3</sup> ] | Young's modulus [GPa] | Poisson's ratio |
|--------------------------------------------------------------------------------|------------------------------|-----------------------|-----------------|
| Aluminum plate                                                                 | 2680                         | 70.6                  | 0.33            |
| Electrode plate                                                                | 6900                         | 100                   | 0.35            |
| Piezoelectric material                                                         | 8400                         | 132                   | 0.30            |
| $\gamma^e = 2.213 \times 10^{-9}$ [F/m], $d_{31} = -3.7 \times 10^{-12}$ [m/V] |                              |                       |                 |

might be expanded as  $h_c$  increased. However, since it was difficult to specify the range in which the circular plate could be regarded as a thin-walled plate,  $h_c$  was set to the above range in order to cover the range of the thin-walled.

The support conditions of the plates, which had flexural rigidity  $D [= Eh_c^3/\{12(1-\nu^2)\}]$ , were expressed by the nondimensional stiffness parameters  $T_n (= Tr_c^3/D)$  and  $R_n (= Rr_c/D)$ . If  $R_n$  ranged from 0 to  $10^{8.0}$  when  $T_n$  was  $10^{8.0}$ , the support condition could be assumed from a simple support to a clamped support. The plate was subjected to the point force  $F_e$  that was set to 1 N and was located at  $r_e/r_c = 0.4$ , as with the excitation experiment.

Fig. 3(a) and (b) show the mechanical power  $P_m$  supplied to the plate by the small vibrator and the electric power  $P_e$ , which are regarded as electricity generation characteristics, as functions of the plate thickness  $h_c$  with changing  $R_n$ . Both of  $P_m$  and  $P_e$  are maximized at  $h_c = 1.0$  mm and decrease rapidly with increasing  $h_c$ . When  $R_n$  increases from  $10^{1.0}$ , the flexural displacement is suppressed due to reinforcing the rigidity of the whole structure, as the result that  $P_m$  and  $P_e$  have a decreasing trend. Although these have the similar tendency, the relative tendency to decrease  $P_e$  with increasing  $h_c$  is somewhat extreme in comparison with that of  $P_m$ . Here, when the plate, whose  $R_n$  is 0 and  $10^{8.0}$ , is subjected to a point force at the plate center, the flexural displacement and bending stress are considered in the statics. The flexural displacement

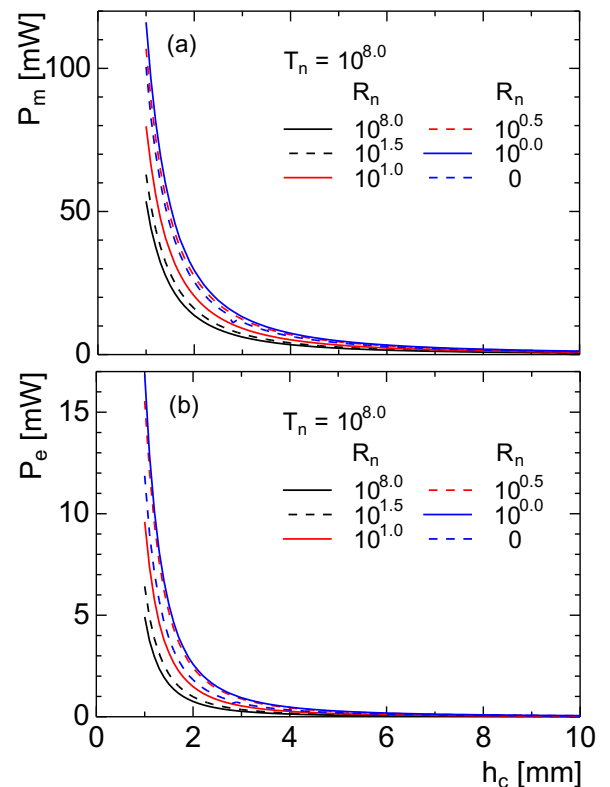


Fig. 3 Variations in (a) mechanical power and (b) electric power with changing plate thickness.

at the center is denoted by  $w_{sm}$  and  $w_{cm}$ , which are derived from  $R_n = 0$  and  $10^{8.0}$ , respectively, and then the bending stress at the center is denoted by  $\sigma_{sm}$  and  $\sigma_{cm}$ . These flexural displacement and bending stress can be calculated as follows:

$$w_{sm} = \frac{3 + \nu_c}{1 + \nu_c} \frac{Fr_c^2}{16\pi D}, \quad (18)$$

$$w_{cm} = \frac{Fr_c^2}{16\pi D}, \quad (19)$$

$$\sigma_{sm} = \frac{F}{h_c^2} \left\{ 0.477 + (1 + \nu_c) \left( 0.485 \ln \frac{r_c}{h_c} + 0.52 \right) \right\}, \quad (20)$$

$$\sigma_{cm} = \frac{F}{h_c^2} (1 + \nu_c) \left( 0.485 \ln \frac{r_c}{h_c} + 0.52 \right). \quad (21)$$

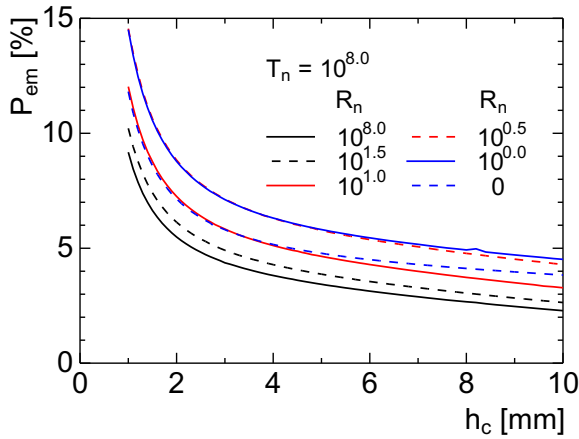


Fig. 4 Electricity generation efficiency as function of plate thickness with changing rotational stiffness.

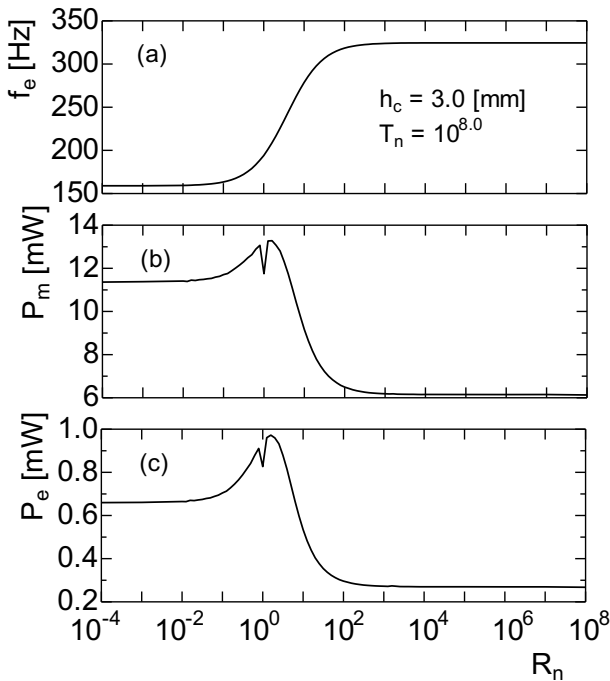


Fig. 5 Variations in (a) natural frequency, (b) mechanical power and (c) electric power with changing rotational stiffness.

$P_m$  depends on  $w_{sm}$  and  $w_{cm}$ , having been obtained from the product  $F$  and  $\dot{w}_c$ . According to Eqs. (18) and (19), a decrease rate  $w_{cm}/w_{sm}$  is constant despite of changes in  $h_c$ , so that the rate of decrease in  $P_m$  is also maintained at the constant ratio. On the other hand,  $P_e$  depends on  $\sigma_{sm}$  and  $\sigma_{cm}$ , having been involved directly with the in-plane strain. The decrease rate  $\sigma_{cm}/\sigma_{sm}$  decreases with increasing  $h_c$ , being calculated actually based on Eqs. (20) and (21). In other words, the rate of decrease in  $P_e$  increases with  $h_c$ . In view of these situations, it is possible to explain a slight difference between variations in  $P_m$  and  $P_e$  with changing  $h_c$ .

In addition, to consider this relationship between  $P_m$  and  $P_e$ , the electricity generation efficiency  $P_{em}$  is defined as follows:

$$P_{em} = \frac{P_e}{P_m} \times 100 [\%]. \quad (22)$$

Because  $P_{em}$  that corresponds to above  $P_m$  and  $P_e$  is indicated in Fig. 4, which has the same manner as Fig. 3, it is natural that  $P_{em}$  behaves in tendency similar to the above powers. However,  $P_{em}$  suppresses considerably the variation rate with  $h_c$  in comparison with those of  $P_m$  and  $P_e$  and is clearly different with changing  $R_n$  in the whole range of  $h_c$ .  $P_{em}$  decreased with increasing  $R_n$  as with  $P_m$  and  $P_e$ , whereas those did not obey that decrease trend, so that the influence of  $R_n$  on electricity generation characteristics is considered in some detail.

Here, the plate of  $h_c = 3.0$  mm is taken up and not only  $P_m$  and  $P_e$  but also its natural frequency  $f_e$  attract attention with changing  $R_n$  in Fig. 5.  $f_e$  is around 160 Hz and is almost constant up to the vicinity of  $R_n = 10^{-0.5}$  from  $R_n = 0$ , which is identical to the simple support and is not exhibited in Fig. 5(a).  $f_e$  increases until around 320 Hz with increasing  $R_n$  after that and is maintained up to  $R_n = 10^{8.0}$ .  $P_m$  is around 11.5 mW and is almost constant up to the vicinity of  $R_n = 10^{-0.5}$  when  $R_n$  increases, as shown in Fig. 5(b). After that,  $P_m$  increases with  $R_n$  and is maximized (13.3 mW) around  $R_n = 10^{0.2}$ . Then  $P_m$  turns to decreases and reaches around 6.15 mW, and then it is maintained up to  $R_n = 10^{8.0}$ . We can make sure that  $P_e$  has the behavior similar to  $P_m$  from Fig. 5(c) and  $P_m$  and  $P_e$  behave in a quite different manner from  $f_e$ .

#### 4.2 Influence of additional force on electricity generation characteristics

In the previous section,  $F_e$  that was derived from the natural frequency  $f_e$  was regarded as the excitation force. Here, a new situation, in which another excitation force  $F_a$  is added to  $F_e$ , is assumed and the excitation frequency of  $F_a$  is denoted by  $f_a$ , and then  $F_a/F_e$  is regarded as the excitation force ratio that is denoted by  $F_R$ . In this case,  $T_n = 10^{8.0}$  and  $R_n = 10^{1.0}$  are adopted to get closer to the experimental support condition. Fig. 6 shows the mechanical power  $P_m$  as functions of  $f_a$  when

$F_R = 1.0$  and  $h_c = 2.0, 3.0$  and  $4.0$  mm.  $P_m$  reaches the respective peaks when  $f_a$  is identical to  $f_e$  and the peak values decrease in order of  $h_c = 2.0, 3.0$  and  $4.0$  mm as with Fig. 3(a). However,  $P_m$  decreases rapidly due to separating  $f_a$  from  $f_e$  no matter what  $h_c$  is.

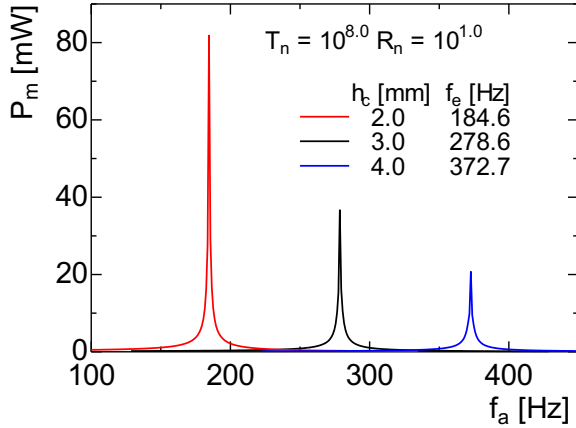


Fig. 6 Mechanical power as function of additional excitation frequency with changing plate thickness.

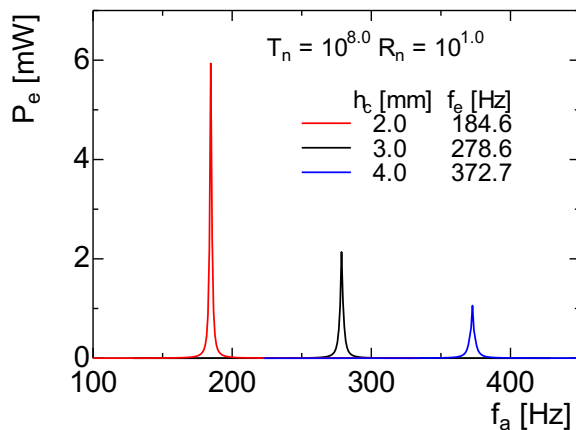


Fig. 7 Electric power as function of additional excitation frequency with changing plate thickness.

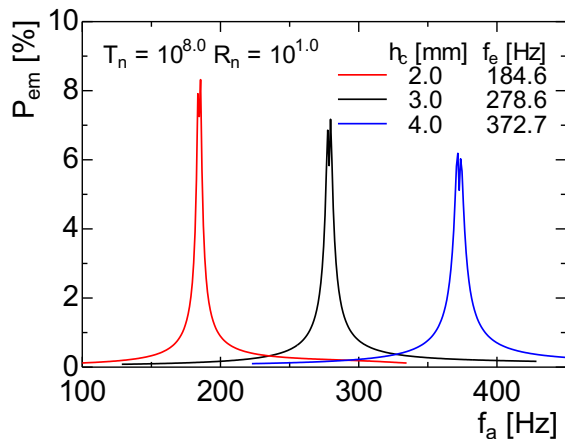


Fig. 8 Electricity generation efficiency as function of additional excitation frequency with changing plate thickness.

$P_e$  is also indicated in Fig. 7, which is shown in the same manner as Fig. 6, and tends to vary similarly to  $P_m$ . However, the rate of decrease in  $P_e$  is greater than that of  $P_m$  and this is because  $P_m$  and  $P_e$  were involved with the flexural displacement and bending stress, respectively, as explained in Eqs. (18) – (21). Specifically, we can make sure of the following relationships by taking notice of Eqs. (19) and (21) based on the clamped support. The rate of variation in the flexural displacement is proportional to the cube of plate thickness ratio and the bending stress is less in the variation rate with the plate thickness ratio than the flexural displacement. However, by taking into consideration the strain energy, which is proportional to the square of bending stress, it is valid that  $P_m$  and  $P_e$  have the above decreasing trend with increasing  $h_c$ . Then Fig. 8 shows the electricity generation efficiency  $P_{em}$ , which was calculated from  $P_m$  and  $P_e$ , as functions of the excitation frequency  $f_a$  of the additional point force  $F_a$ . The rate of variation in  $P_{em}$  with  $h_c$  is moderated in

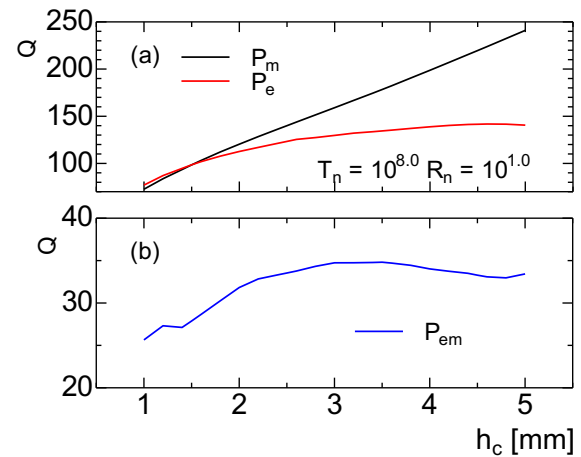


Fig. 9 Variations in  $Q$  values with changing plate thickness: (a) mechanical and electric powers and (b) electricity generation efficiency.

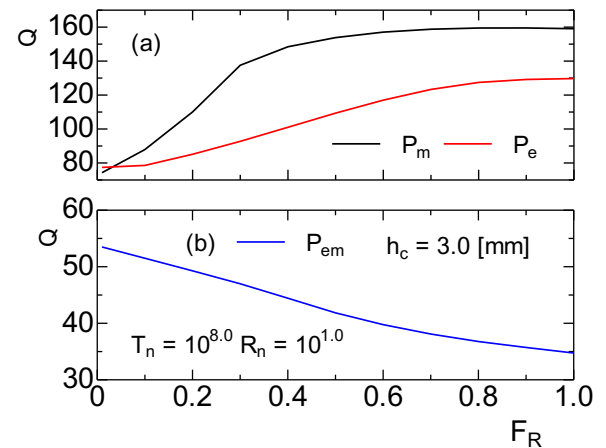


Fig. 10 Variations in  $Q$  values with changing excitation force ratio: (a) mechanical and electric powers and (b) electricity generation efficiency.

comparison with those of  $P_m$  and  $P_e$  which were relatively close, because  $P_{em}$  was obtained from dividing  $P_e$  by  $P_m$ . In order to consider not only the respective values but also generation situations of the respective peaks,  $Q$  value was adopted and was defined as follows:

$$Q = \frac{f_e}{f_2 - f_1}, \quad (23)$$

where  $f_2 - f_1$  is half-value width with respect to the respective peaks in Figs. 6 – 8.

Fig. 9 shows such a  $Q$  value as functions of  $h_c$  on condition that  $P_m$  and  $P_e$  (Fig. (a)) and  $P_{em}$  (Fig. (b)) are indicated by means of separating figures, because  $Q$  in  $P_m$  and  $P_e$  is greater and varies in the more extensive range than that of  $P_{em}$ . In general,  $Q$  value is a dimensionless quantity to express a state of vibration system and is the quantity dividing energy stored in the vibration system by energy dissipated from the system in a period. In other words, this value is also index to express the sharpness of peaks and such a function was utilized here. Although the natural frequency  $f_e$  was proportional to  $h_c$ , since  $Q$  in  $P_m$  also increases linearly with  $h_c$ , it may look like promoting the sharpness of  $P_m$  with increasing  $h_c$ . Although the variation rate  $f_e / h_c$  was around 94.0,  $Q / h_c$  is around 42.1. In order to make  $Q$  behave like  $f_e$ ,  $Q / h_c$  needs to be increased up to around 73.5, and that value is based on the assumption that the half width at  $h_c = 1.0$  mm is maintained in the whole range of  $h_c$ . However, because  $Q / h_c$  is much smaller than not only  $f_e / h_c$  but also 73.5, actually, the sharpness of  $P_m$  peaks becomes dull gradually with increasing  $h_c$ .

Although  $Q$  in  $P_e$  increases with  $h_c$  as with that in  $P_m$ , its increase rate decreases gradually with increasing  $h_c$ . In other words, the sharpness of  $P_e$  peaks becomes dull more rapidly with increasing  $h_c$ . On the other hand,  $Q$  in  $P_{em}$  ranges from 25 to 35 and distributes in the lower range than those in  $P_m$  and  $P_e$ , so that the sharpness of  $P_{em}$  peaks becomes duller than that of  $P_m$  and  $P_e$  peaks. Then changes in  $Q$  of  $P_{em}$  are suppressed in comparison with those of other cases, in particular, its  $Q$  turns to a decreasing trend beyond  $h_c = 3.5$  mm and the situation to dull the sharpness of  $P_{em}$  peaks is promoted more strongly. The tendencies mentioned above correspond to variation tendencies of the peaks in Figs. 6 – 8.

Up to here, the original point force  $F_e$  was coincident with the additional point force  $F_a$ , i.e., the excitation force ratio  $F_R$  that is 1.0 was regard as the analysis condition. Therefore, shifts in  $Q$  values with respect to  $P_m$ ,  $P_e$  and  $P_{em}$  herein were considered when  $h_c = 3.0$  mm and the  $F_R$  range of 0.01 to 1.0.  $Q$  in  $P_m$  and  $P_e$  is shown in Fig. 10(a) and  $Q$  in  $P_{em}$  is shown in Fig. 10(b). In these cases, since  $f_e$  is set to 278.6 Hz and is constant, the consideration against changes in  $f_e$ , which was taken in Figs. 6 – 8, is ignored. Although  $Q$  in  $P_m$  increases greatly with  $F_R$  up to  $F_R \approx 0.35$ , its increase is weakened after

that and it converges to the vicinity of 159. Also increasing with  $F_R$ ,  $Q$  in  $P_e$  is smaller than that of  $P_m$  and behaves more monotonously.

$Q$  in  $P_{em}$  decreases gradually with increasing  $F_R$  and presents a totally reverse tendency against those in  $P_m$  and  $P_e$ . This takes place in the lower range of  $Q$  as well as that of changes in  $h_c$ , so that the peaks of  $P_{em}$  become duller with increases in  $h_c$  and  $F_R$  than those of  $P_m$  and  $P_e$ . In other words, it is suggested that  $P_{em}$  becomes more insensitive with increases in additional forces than  $P_m$  and  $P_e$ .

#### 4.3 Electricity generation efficiency for random excitation

In previous section, the electricity generation characteristics attracted attention on condition that the plate was subjected to two point forces, whose frequencies were different. Considering the actual vibration environment, we want to aim to apply random excitation to this electricity generation system. However, the application of the random excitation to the analytical model was hard in reason why our computer performance lacked in calculation capacity and speed. Therefore, the electricity generation efficiency  $P_{em}$  under the random excitation was estimated from the experimental results.

$P_{em}$  that was obtained from the electricity generation experiment in Fig. 2(a) is indicated with changes in the plate thickness  $h_c$  by black circle and line in Fig. 11(a).  $P_{em}$  decreases with increasing  $h_c$  and has the tendency similar to theoretical value of  $T_n = 10^{8.0}$  and  $R_n = 10^{1.0}$  in Fig. 4, being smaller than theoretical value that did not take into consideration the acoustic radiation derived from the plate vibration. Then a part energy of such an acoustic radiation can be scavenged by causing air column resonance within a cavity. Specifically, mechanical-acoustic coupling is caused not only between the plate vibration on the excitation side and the internal acoustic field but also between the plate vibration on the non-excitation side and the internal acoustic field, as shown in Fig. 2(b). In actual experiment, the end plates of the same thicknesses were chosen and then cylinder length  $L$  at which coupling is promoted was set. Such a  $P_{em}$  is drawn by red circle and line in Fig. 11(a) and presents a totally reverse tendency against the above results without the cavity, as the result that we can confirm the improvement of electricity generation characteristics by means of coupling.

The results based on applying the random excitation to the plate of Fig. 2(a) are shown by blue circle and line in Fig. 11(b). Although  $P_{em}$  of the random excitation decreases gradually with increasing  $h_c$  and has the behavior similar to that of the excitation by the natural frequency, its values are suppressed more extremely than those of the excitation by the natural frequency. However, when  $h_c = 3.0$  mm in Fig. 8, if it

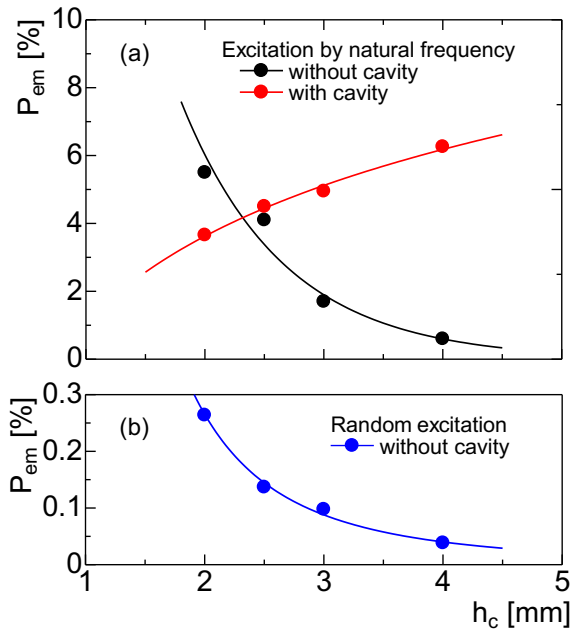


Fig. 11 Variations in electricity generation efficiency with changing plate thickness: (a) harmonic excitation and (b) random excitation.

is considered that  $P_{em}$  was  $8.19 \times 10^{-2}$  and 0.167 % at  $f_a = 128.6$  and 428.6 Hz, which are the upper and lower limits in the analytical frequency range of  $f_a$ , respectively, it is realized that these experimental results are valid. In other words, the additional excitation, whose frequency goes away from the natural frequency, gets closer rapidly to the situation of the random excitation. The random excitation was applied to the experimental apparatus of  $h_c = 3.0$  mm with the cavity in Fig. 2(b) and its  $P_{em}$  is shown with changes in the cylinder length  $L$  in Fig. 12. However, the coupling effect that was exhibited in Fig. 11(a) is not verified from this result, so that a new method to improve electricity generation characteristics needs to be attempted.

## 5. Conclusion

In this study, an electricity generation system, which consisted of a circular plate with a piezoelectric element installed at its center, was examined. The plate was subjected to the point force of the natural frequency, which was derived from the plate material, dimensions and support condition, in the case of the original application. To assume the application of this electricity generation system to actual environment, the point force, whose frequency was different from the natural frequency, was added to the original point force. Then the excitation force ratio, which was determined by dividing the

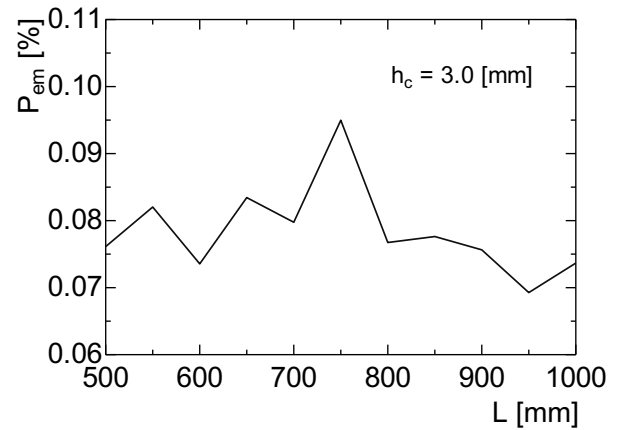


Fig. 12 Electricity generation efficiency as functions of cylinder length under random excitation.

additional point force by the original point force, was defined. The mechanical power supplied to the plate, the electric power in the electricity generation and the electricity generation efficiency derived from those powers were considered as electricity generation characteristics from theoretical analysis under such an excitation condition. As a result, the respective values of electricity generation characteristics decreased rapidly with changes in the additional excitation frequency that went away from the natural frequency. Then it was verified that such an additional excitation got closer to the situation of the random excitation from the electricity generation experiment, and then from the estimation based on  $Q$  value, the electricity generation efficiency became more insensitive against shifts in the additional excitation frequency than the above two powers and also became insensitive with increases in the plate thickness and excitation force ratio.

To improve the electricity generation characteristics by using the acoustic radiation power derived from the plate vibration, a cylinder that had the above plates at both ends was adopted, and the mechanical-acoustic coupling between the plate vibrations and internal sound field was used. In the case of the excitation by the natural frequency, the improvement of the electricity generation efficiency by coupling was promoted with increasing the plate thickness. However, although the application of the random excitation to this system suppressed the electricity generation characteristics more extremely than those of the harmonic excitation, the characteristics were hardly improved only by mechanical-acoustic coupling.

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